AUS | American University of Sharjah

Score:

American University of Sharjah
Department of Mathematics and Statistics

Final Exam - spring 2018
MTH 111 – Math for Architects

Instructor Name: **Ayman Badawi**

➔The name above *must be* the name of *your* instructor🡐

SCORE (---98---)
100

Student Name: NADIN ELSHIRBINI
Student ID Number: 72434

1. *No Questions are allowed during the examination.*
2. *This exam has 6 pages plus this cover page .*
3. *Do not separate the pages of the exam.*
4. *Scientific calculator are allowed but cannot be shared. Graphing Calculators are not allowed.*
5. *Take off your cap. Turn off all cell phones and remove all headphones.*
6. *No communication of any kind is permitted.*
7. *All working must be shown*

Student signature: ______________________________

Final Exam: MTH 111, Spring 2018

Ayman Badawi

Points = $\frac{\quad}{100}$

QUESTION 1. (9 points) Find y' and DO NOT SIMPLIFY

(i) $y = (x+1)e^{(3x+2)}$

$y' = e^{3x+2} + (3x+3)e^{3x+2} = e^{3x+2}(3x+4)$ ✓

(ii) $y = ln[(3x-2)^4(2x+1)^7]$

$y' = \frac{12}{3x-2} + \frac{14}{2x+1}$ ✓

(iii) $y = (7x+2)^9$

$y' = 63(7x+2)^8$ ✓

QUESTION 2. (i) **(6 points)** Does the line line $L_1 : x = t+1, y = t-1, z = 7$ intersect the line $L_2 : x = -w+4, y = w-2, z = 2w+3$? If yes, then find the intersection point. Is L_1 perpendicular to L_2?

$D_1 < 1,1,0 >$ $D_2 < -1,1,2 >$

$D_1 \neq c D_2 \Rightarrow$ $\boxed{L_1 \text{ and } L_2 \text{ intersect}}$

$D_1 \times D_2 = < 2,-2,2 >$ $\Rightarrow L_1$ not $\perp L_2$.

-2

$t+1 = -w+4 \to t+w = 3$

$t-1 = w-2 \to t-w = -1$

$t = 1 \quad w = 2$

Using $t=1$:
$x = 1+1 = 2$
$y = 1-1 = 0$
$z = 7$

or

Using $w=2$:
$x = -2+4 = 2$
$y = 2-2 = 0$
$z = 2(2)+3 = 7$

$\boxed{\text{point of intersection } (2,0,7)}$ ✓

(ii) **(4 points)** Convince me that $L_1 : x = t, y = 10, z = -t+4$ is parallel to $L_2 : x = 4w+1, y = 7, z = -4w+2$

$D_1 = <1,0,-1>$ $D_2 = <4,0,-4>$

$D_2 = 4D_1$

$t=0 \to (0,10,4)$

$\left.\begin{array}{l} x:\ 0 = 4w+1 \to w = -\frac{1}{4} \\ z:\ 4 = -4w+2 \to w = -\frac{1}{4} \\ y:\ w = 0 \end{array}\right\}$ diff. $w \Rightarrow L_1$ and L_2 are parallel. ✓

(iii) Let $Q_1 = (1, 1, 0)$, $Q_2 = (0, -1, 2)$ and $Q_3 = (2, 2, 2)$.

a. **(5 points)** Find the equation of the plane that contains Q_1, Q_2, Q_3.

$\overrightarrow{Q_1Q_2} < -1, -2, 2 > \quad \overrightarrow{Q_1Q_3} < 1, 1, 2 >$

$N = |Q_1Q_2 \times Q_1Q_3| = \begin{vmatrix} i & j & k \\ -1 & -2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = < -6, 4, 1 >$

$P: \; -6(x-2) + 4(y-2) + 1(z-2) = 0$

b. **(2 points)** Find the area of the triangle that has Q_1, Q_2, Q_3 as vertices.

$A = \frac{1}{2} |\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}| = \frac{\sqrt{6^2+4^2+1^2}}{2} = \frac{\sqrt{53}}{2}$ units^2

(iv) **(4 points)** Given $L: x = t+1, y = 8, z = 4t+1$ lies entirely inside the plane $P: ax + 2y + z = b$ Find the values of a, b. $D < 1, 0, 4 > \quad N < a, 2, 1 >$

$N \cdot D = 0$

$a + 4 = 0$

$\boxed{a = -4}$

$-4(t+1) + 2(8) + 4t + 1 = b$

$-4t - 4 + 16 + 4t + 1 = b$

$\boxed{b = 13}$

(v) **(4 points)**item Find the distance between the point $(1, -1, 1)$ and the line $L: x = t+1, y = 2t+3, z = -2t+10$

$Q(1,-1,1) \quad I(1,3,10)$

$V = \overrightarrow{IQ} = < 0, -4, -9 > \quad D < 1, 2, -2 >$

$V \times D = \begin{vmatrix} i & j & k \\ 0 & -4 & -9 \\ 1 & 2 & -2 \end{vmatrix} = < 26, -9, 4 >$

$d = \frac{|V \times D|}{|D|} = \frac{\sqrt{26^2+9^2+4^2}}{\sqrt{1^2+2^2+2^2}} = \frac{\sqrt{773}}{3}$ units

(vi) **(3 points)** For what values of x will the tangent line to the curve $f(x) = e^x - 4x + 2$ be horizontal? (Hint: Note that horizontal lines have slope 0)

$f'(x) = e^x - 4$

$0 = e^x - 4$

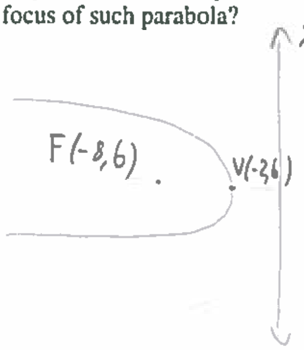
$e^x = 4$

$\ln e^x = \ln 4$

$x \ln e = \ln 4$

$\boxed{x = \ln 4}$

(vii) **(5 points)**Find the equation of a parabola that has x = 4 as its directrix line and (-2, 6) as its vertex. What is the focus of such parabola?



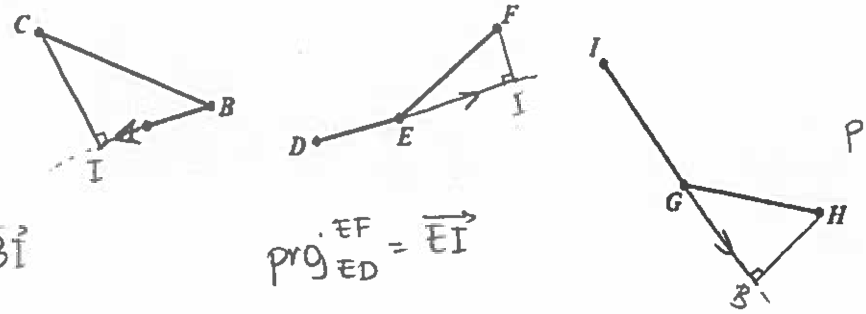
$d = |-2-4| = 6$

$-4d\,(x - x_0) = (y - y_0)^2$

$\boxed{-24(x+2) = (y-6)^2}$

$\boxed{F(-8,6)}$

(viii) (6 points)



proj$_{BA}^{BC}$ = $\overrightarrow{BI}$

proj$_{ED}^{EF}$ = $\overrightarrow{EI}$

proj$_{GI}^{GH}$ = $\overrightarrow{GB}$

Use the pictures above
1. Draw the projection vector of BC over BA
2. Draw the projection vector of EF over ED
3. Draw the projection vector of GH over GI

(ix) Let $f(x) = (x^2 - 6x + 5)^4$.

a. (3 points)Find $f'(x)$. Then find the sign of $f'(x)$.

$f'(x) = 4(2x-6)(x^2-6x+5)^3$
$0 = 4(2x-6)(x^2-6x+5)^3$
$2x-6=0$ $\quad$ $x^2-6x+5=0$
$x=3$ $\quad$ $x=5$ $\quad$ $x=1$

x	$-\infty$		1		3		5		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		↘		↗		↘		↗	

$f'(x)$ negative for $(-\infty, 1) \cup (3,5)$
$f'(x)$ positive for $(1,3) \cup (5,+\infty)$

b. (2 points)For what values of x does $f(x)$ increase?

$(1,3) \cup (5,+\infty)$

c. (2 points)For what values of x does $f(x)$ decrease?

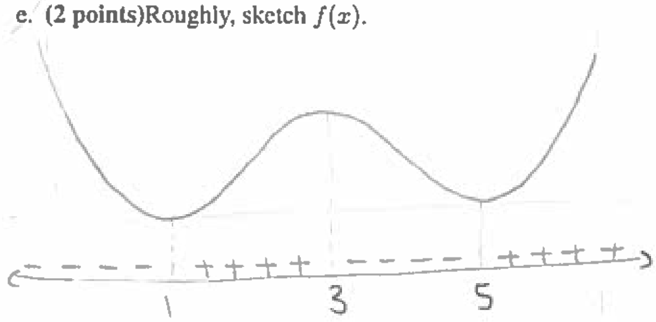
$(-\infty,1) \cup (3,5)$

d. (2 points)Find all local min (max) points of $f(x)$ if possible

min at $x=1$ and $x=5$
max at $x=3$

MIN : $(1,0)$ and $(5,0)$
MAX : $(3,256)$

e. (2 points)Roughly, sketch $f(x)$.



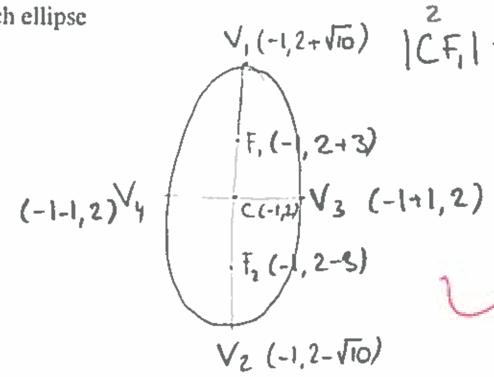
(x) Consider the ellipse $(x+1)^2 + \frac{(y-2)^2}{10} = 1$

$C(-1,2)$

$\frac{k}{2} = \sqrt{10}$

$|CF_1| = \sqrt{10-1} = 3$

a. **(2 points)** Roughly, draw such ellipse



b. **(2 points)** Find the foci

$F_1(-1,5)$

$F_2(-1,-1)$

c. **(2 points)** Find the ellipse constant

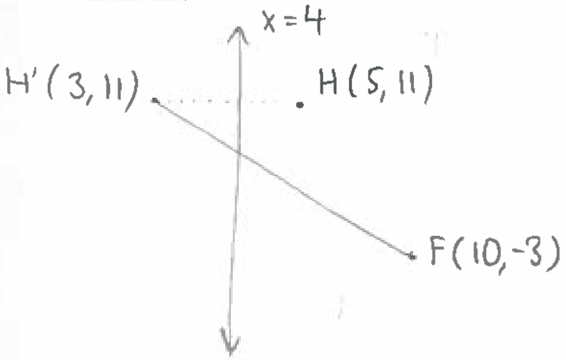
$k = 2\sqrt{10}$

d. **(2 points)** Find all four vertices

$V_1(-1, 2+\sqrt{10})$ $V_3(0,2)$

$V_2(-1, 2-\sqrt{10})$ $V_4(-2,2)$

(xi) **(6 points)** Let $H = (5, 11)$ and $F = (10, -3)$. Find a point Q on the vertical line $x = 4$ such that $|HQ| + |QF|$ is minimum.



$m = \frac{-3-11}{10-3} = -2$

$11 = -2(3) + b$

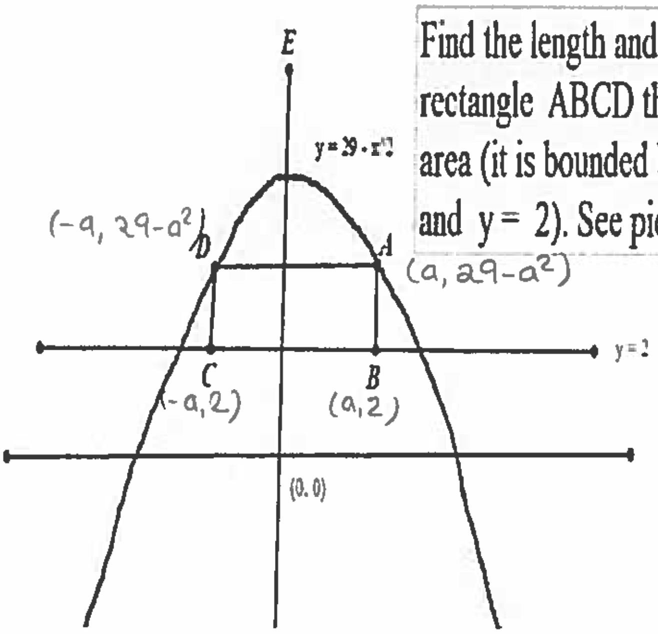
$b = 17$

$y = -2x + 17$

$y = -2(4) + 17 = 9$

$Q(4,9)$

(xii) **(8 points)**



Find the length and the width of the rectangle ABCD that has maximum area (it is bounded by y = 29 - x^2 and y = 2). See picture

$W = |BC| = 2a$

$L = |AB| = 29 - a^2 - 2 = 27 - a^2$

$A = LW = 2a(27 - a^2)$

$A = 54a - 2a^3$

$A' = 54 - 6a^2$

$0 = 54 - 6a^2$

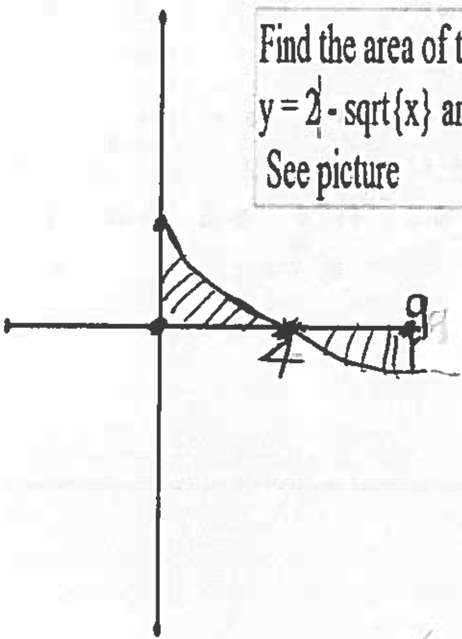
$54 = 6a^2 \Rightarrow a = 3$

$A'' = -12a \Big|_{a=3} < 0 \Rightarrow$ max. at $a = 3$

$W = 2a = 6$ units

$L = 27 - a^2 = 18$ units

(xiii) **(6 points)**



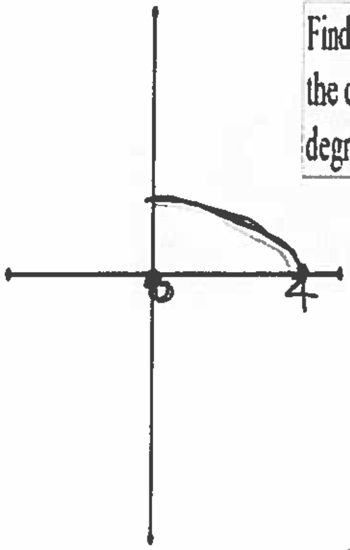
Find the area of the shaded region that is bounded by y = 2 - sqrt{x} and x-axis, where x is between 0 and 9. See picture

$A = \int_0^9 2 - \sqrt{x}\, dx = \int_0^4 2 - \sqrt{x}\, dx - \int_4^9 2 - \sqrt{x}\, dx$

$= \left[2x - \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 \right] - \left[2x - \frac{2}{3} x^{\frac{3}{2}} \Big|_4^9 \right]$

$= \frac{8}{3} - \left(0 - \frac{8}{3}\right) = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$ unit^2

(xiv) **(4 points)**



Find the volume of the solid object that is obtained by rotating the curve of y = sqrt{4 - x}, where x is between 0 and 4, 360 degrees about the x-axix

$$V = \pi \int_0^4 (\sqrt{4-x})^2\,dx = \pi \int_0^4 4-x\,dx$$

$$= \pi \left(4x - \frac{x^2}{2} \Big|_0^4 \right) = \pi (8-0)$$

$$= 8\pi \text{ units}^3$$

(xv) **(3 points)** $\frac{1}{6}\int 6x^2(2x^3+7)^9\,dx$

$$\frac{(2x^3+7)^{10}}{60} + C$$

(xvi) **(3 points)** $\int \frac{(x+1)}{x^2+2x+3}\,dx$

$$\frac{\ln|x^2+2x+3|}{2} + C$$

(xvii) **(3 points)** $\frac{1}{4}\int 4(x+5)e^{(2x^2+20x+1)}\,dx$

$$\frac{1}{4}e^{2x^2+20x+1} + C$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com